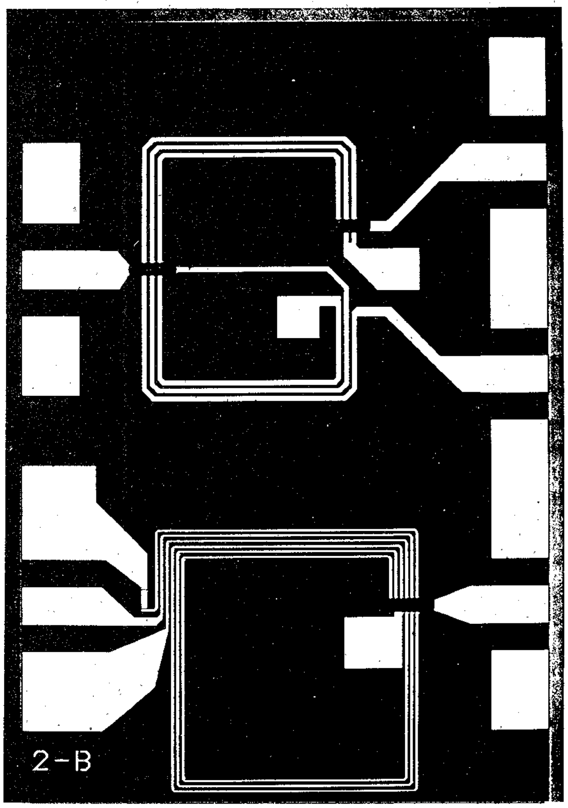

Fig. 3. Photograph of MMIC triformers.

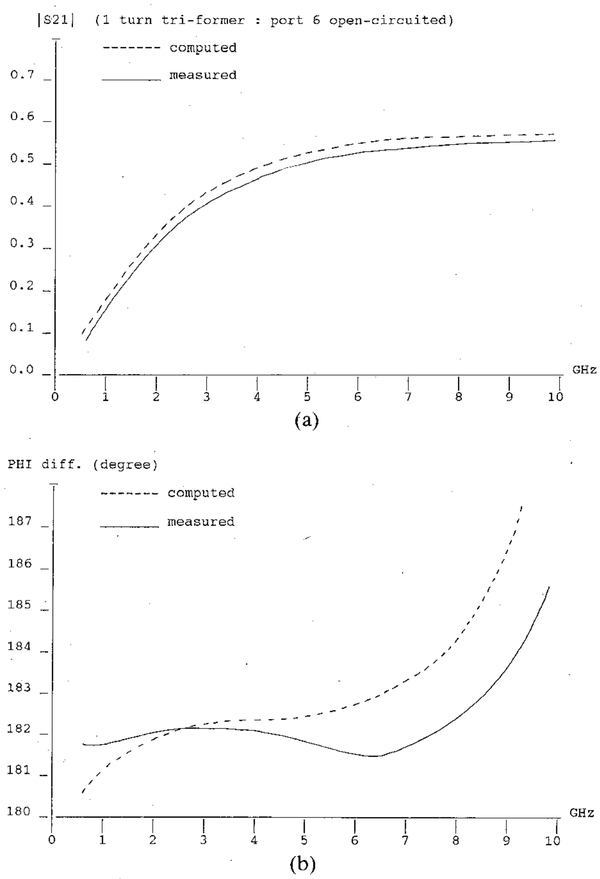

Fig. 4. Comparison of computed and measured S parameters. (a) Magnitude. (b) Differential phase shift.

over the frequency range 1–10 GHz. The application of these devices as wide-band baluns for MMIC balanced mixers is foreseen.

Acknowledgment

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Analysis of Edge-Coupled Elliptical (Oval) Rods Between Infinite Ground Planes

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***Abstract* —The paper presents an analysis of the even- and odd-mode impedances for a generalized edge-coupled structure consisting of two elliptical (oval) conducting rods. Data on the even- and odd-mode impedances for different special cases of the generalized elliptical (oval) structure are presented. Some special cases of the present formulation are compared with results available in the literature.**

I. Introduction

The analysis of parallel edge-coupled strips has been presented in the literature by Cohn [1]. Wheeler analyzed the transmission properties of a single round wire between two parallel planes [2].

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There exist some studies on coupled structures consisting of circular rods [3], but these were carried out with the coupled circular rods loaded periodically between the infinite ground planes. Such a periodically loaded structure may find application in filter circuits. Subsequently in a short communication Cristal presented data for partially decoupled round rods between parallel ground planes [4]. Chisholm analyzed trough and slab lines useful for the estimation of the odd-mode impedance of coupled circular bars [5]. Later Levy analyzed a system of two coupled circular conductors using conformal transformation with numerical techniques [6]. Recently Rao analyzed a transmission structure having an oval-shaped center conductor located between infinite parallel planes [7], inside a box [8], and between finite ground planes enclosed by magnetic walls [9]. But these studies were limited to a single center conductor only. Currently, no data are available on an edge-coupled system with generalized elliptical conducting bars. This communication is aimed at filling this gap. The even- and odd-mode impedances of a system of two edge-coupled elliptical (oval) rods are determined from an evaluation of the capacitance of the basic structure, which has been dealt with earlier [7]–[9]. However it needs to be pointed out that this oval structure closely approximates a generalized elliptical structure up to a certain impedance values of the line. For a clear definition and details the reader is referred to some of the earlier references [7]–[9]. The structures with rounded center conductor are useful for high-power applications. As there are no discontinuities in the center conductor as in conventional strips, higher order modes will not be excited. This will therefore reduce the losses. Apart from this it should be emphasized that this formulation is useful in the design of various couplers, including edge line couplers useful for high-power applications. Another important aspect is that when the eccentricity of the generalized elliptical structure is close to unity, the configuration corresponds to the case of a system of conductors having finite width with rounded corners. This practical structure is useful for switches with high power handling capabilities. Sample data for some compression ratios of the ellipse that cover the various cases are presented, with some comparisons.

II. Calculation of Even- and Odd-Mode Impedances

The analyzed structure, consisting of two edge-coupled elliptical (oval) conducting bars between infinite parallel planes, is shown in Fig 1. For the even-mode impedance case, where the excitation of the two center conductors is in phase, there appears a fictitious magnetic wall exactly in the middle of the conductors perpendicular to the ground plane. Hence the even-mode impedance can be evaluated from a determination of the capacitance of the structure with this magnetic wall (Fig. 1(b)). But determination of the capacitance of this structure is not straightforward. However, this structure can be decomposed into two simple basic structures connected in parallel as shown in Fig. 1(b). Interestingly, these two basic structures are dealt with in the literature; hence the capacitance is simply obtained as the sum of the capacitances of the basic structures indicated in Fig. 1(b). Similarly the odd-mode impedance is evaluated by replacing the magnetic wall by an electric wall (Fig. 1(c)). The capacitances of the basic structures have been obtained from equations in earlier papers [7]–[9], and are reproduced here in Table I for the sake of completeness. These equations are derived using conformal transformation and are exact for the oval configuration, and the range of validity of these equations to an exact elliptical conductor is discussed in earlier references [7]–[9]. From the configurations with magnetic and electric walls, the even- and odd-mode capaci-

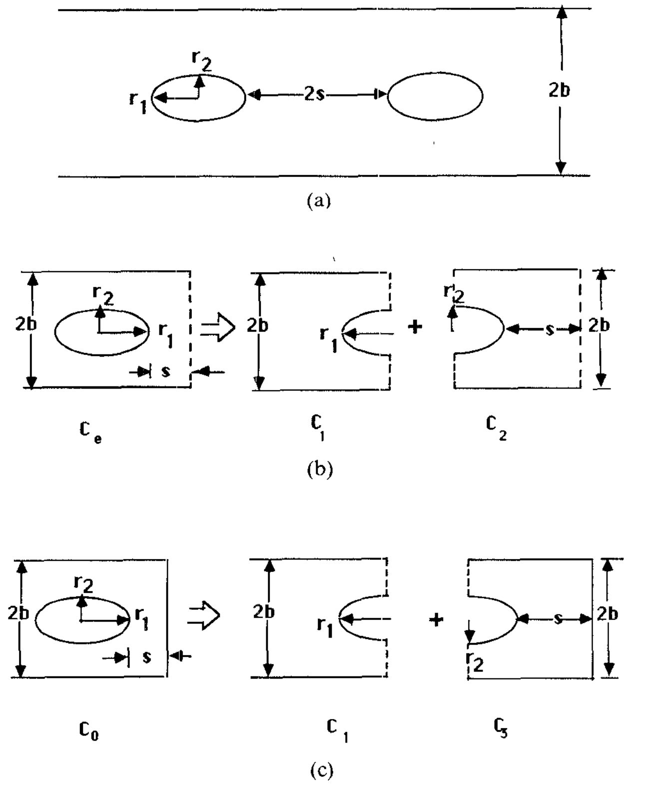
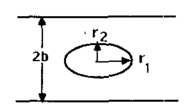
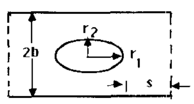
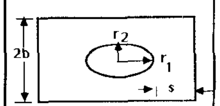


Fig. 1. (a) Configuration of transmission structure with two edge-coupled elliptical bars (b) Component capacitances of the even-mode configuration (c) Component capacitances of the off-mode configuration.

TABLE I
Equations for the Capacitance of the Basic Structures

	$C_1 = 2\epsilon_0\epsilon_r \frac{K(n)}{K'(n)}$	(T1)
	$\frac{r_1}{b} = \frac{2}{\pi(1+\lambda_1)} \ln\frac{(1+n)}{(1-n)}$	(T2)
	$\frac{r_2}{b} = \frac{2\lambda_1}{\pi(1+\lambda_1)} \sin^{-1}(n)$	(T3)
$C_T = 2C_1$	$\lambda_1 = \frac{\ln\frac{(1+n)}{(1-n)}}{k \sin^{-1}(n)}$	(T4)
	$k = \frac{r_1}{r_2}$	(T5)
	$C_2 = 2\epsilon_0\epsilon_r \frac{K(p)}{K'(p)}$	(T6)
	$C_3 = 2\epsilon_0\epsilon_r \frac{K(\alpha^2)}{K'(\alpha^2)}$	(T7)
	$\frac{r_1}{b} = \frac{\lambda\sqrt{1-p}F(\sin^{-1}\alpha\|m)}{K(g)+\lambda\sqrt{1-p}K'(m)}$	(T8)
$C_T = 2C_2$	$\frac{r_2}{b} = \frac{K(g)-F(\sin^{-1}\sqrt{1-p}\|g)}{K(g)+\lambda\sqrt{1-p}K'(m)}$	(T9)
	$\frac{s}{b} = \frac{K(g)+\lambda\sqrt{1-p}[K(m)-F(\sin^{-1}\alpha\|m)]}{K(g)+\lambda\sqrt{1-p}K'(m)}$	(T10)
	$\lambda = k\frac{K(g)-F(\sin^{-1}\sqrt{1-p}\|g)}{\sqrt{1-p}F(\sin^{-1}\alpha\|m)}$	(T11)
	$p = m\alpha^2 \quad g = \frac{(1-m)}{(1-p)}$	(T12)
$C_T = 2C_3$	F is incomplete elliptic integral with modulus m, K and K′ respectively correspond to complete elliptic integral with modulus m and (1-m). α is a parameter and should be selected arbitrarily with in the range of 0 to 1.	

* C_T denotes Capacitances of respective Structures

tances of the equivalent even- and odd-mode structures can be written as

$$C_e = C_1 + C_2 \tag{1}$$

$$C_o = C_1 + C_3 \tag{2}$$

where C_1, C_2, and C_3 are the capacitances per unit length of the structures in Fig. 1. The even- and odd-mode impedances Z_{0e} and Z_{0o} are obtained in terms of C_e and C_o from the following simple relations:

$$Z_{0e} = \frac{120\pi}{\sqrt{\epsilon_r}\, C_e} \tag{3}$$

$$Z_{0o} = \frac{120\pi}{\sqrt{\epsilon_r}\, C_o} \tag{4}$$

where ϵ_r is the relative dielectric constant of the medium surrounding the center conductors.

The calculations of the even- and odd-mode impedances were made in the following way (refer to equations in Table I):

Step 1) Choose a value of the parameter α in reasonable steps ($0. < \alpha < 1.$) and calculate C_3 from equation T7.

Step 2) For each chosen value of α^2, given S/b and the compression ratio of the ellipse $k(=r_1/r_2)$, solve the set of transcendental equations (T10, T11, and T12) from Table I numerically to obtain the values of m, p, and g.

Step 3) From the values of α^2, m, p, and g, calculate $r_1/b, r_2/b$, and C_2 from equations T8, T9, and T6 respectively.

Step 4) Using these values of $r_1/b_2, r_2/b$, and k, solve the set of transcendental equations (T2,T3,T4, and T5) to obtain the value of n. Calculate C_1 from equation T1.

Step 5) Obtain Z_{0e} and Z_{0o} from equations (1)–(4).

III. Results and Discussions

Following the steps outlined in the previous section, the even- and odd-mode impedances were calculated for five different edge-coupled configurations and the results of these calculations are presented in Figs. 2–6. It was found that the results obtained for the coupling structure consisting of two parallel strips (Fig. 2) exactly correspond to those of Cohn, as the transformation equations of the present formulation for this case are identical to the transformation derived by Cohn [1]. It was also observed that Wheeler's results for a single circular conductor [2] agree well with the present analysis for the case of circular conducting bars when the spacing between the conductors is large ($S/b > 2.0$). In addition, the results for the even- and odd-mode impedances for the case of circular conducting bars are compared with Levy's in Table II. The results of these two formulations agree to within 5 percent. This deviation is due to the fact that the present structure is an oval and starts deviating from the circular shape for higher values of r/b. However the analysis is exact for the limiting cases of conductors with horizontal as well as vertical center strips. From the results of Figs. 2–6, it is observed that the deviation between the even- and the odd-mode impedance increases with decreasing values of the compression ratio. This deviation is maximum for the case of the edge line edge coupler. For a given spacing the deviation between the even- and the odd-mode impedance is a measure of the coupling.

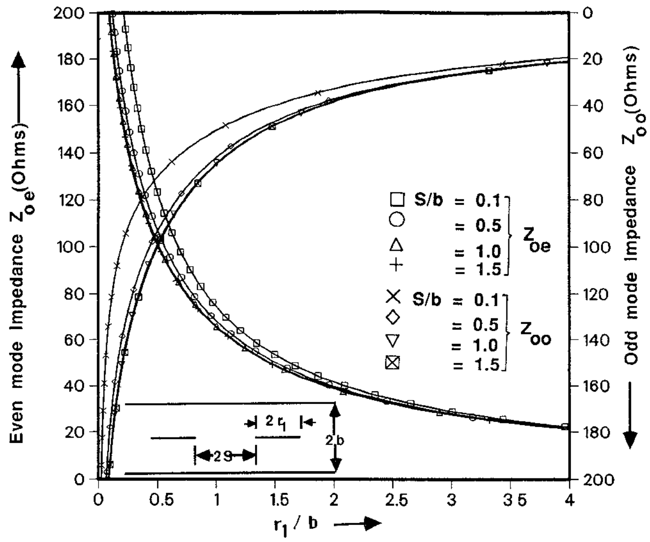


Fig. 2. Variation of even- and odd-mode impedances as a function of r_1/b for $k=\infty$. *All these results exactly match with those of Cohn [1] for this case of oval.

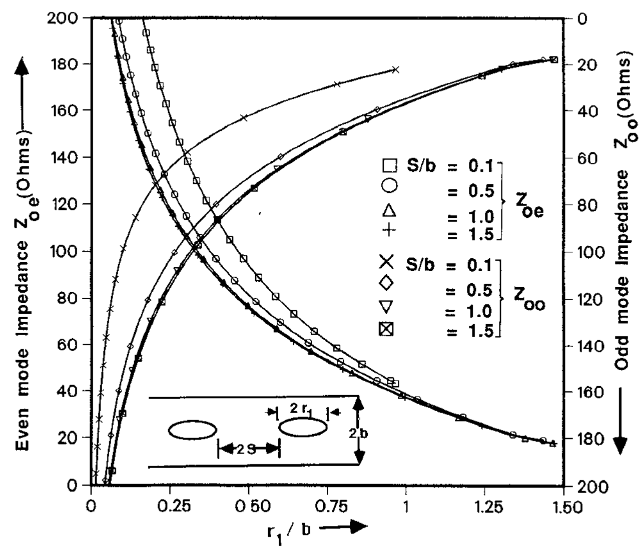


Fig. 3. Variation of even- and odd-mode impedances as a function of r_1/b for $k=2.0$.

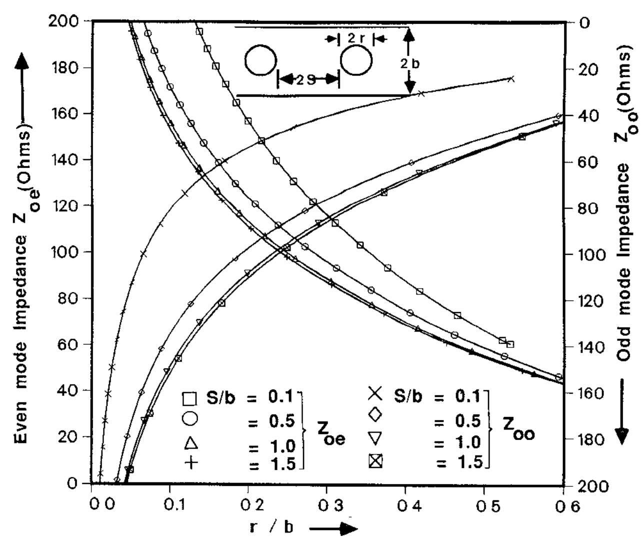


Fig. 4. Variation of even- and odd-mode impedances as a function of r/b for $k=1.0$.

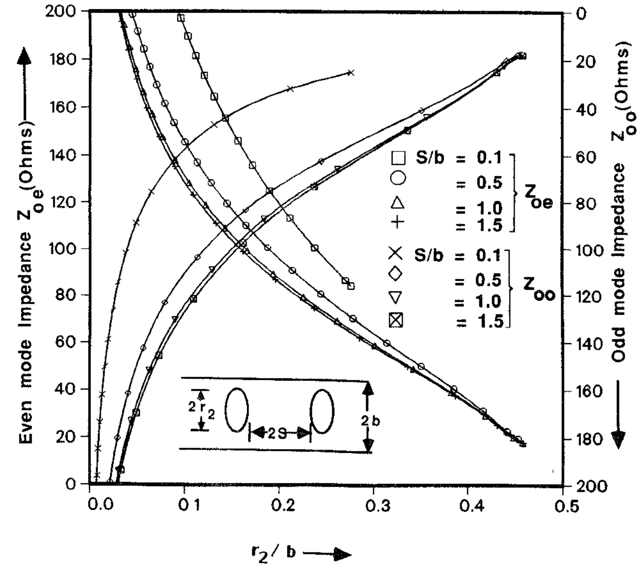


Fig 5. Variation of even- and odd-mode impedances as a function of r_2/b for $k = 0.5$.

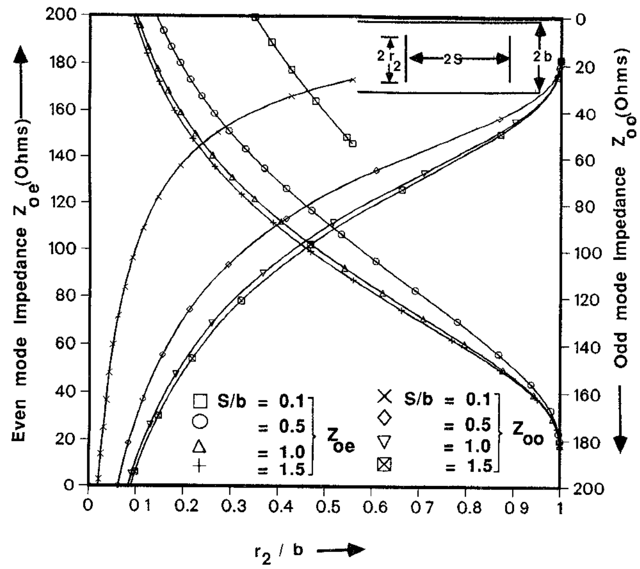


Fig. 6. Variation of even- and odd-mode impedances as a function of r_2/b for $k = 0\,0$

IV. Conclusions

Data on a generalized structure of elliptical (oval) conductors have been presented from the available analysis of some of the basic transmission structures. With a single set of equations, different shapes of the center conductors of the coupling structures can be analyzed by choosing the parameter α. Close agreement of the results of the present formulation with some special cases available in the literature confirms the validity of the analysis. This formulation can also be used to find the impedance of a transmission structure having a center conductor of finite thickness with rounded corners by choosing a low value of the compression ratio. Such structures are useful for high-power applications.

TABLE II
Comparison between the Even- and Odd-Mode Impedances Obtained from Levy's Method and the Present Work for the Case of Coupled Circular Bars

		Z_{oe}(ohm)			Z_{oo}(ohm)		
r/b	S/b	Levy	present work	%error	Levy	present work	%error
.354	.176	96.2867	95.6461	0.6653	49.9141	48.2222	3.3896
.400	.200	84.8239	84.1968	0.7393	47.9315	46.4332	3.1259
.400	.226	83.6271	82.9573	0.7985	49.9962	48.6779	2.6368
.436	.280	74.9187	74.2094	0.9468	49.9320	48.9281	2.0105
.462	.338	68.8971	68.2559	0.9307	50.0181	49.2143	1.6070
.482	.398	64.5025	63.8566	1.0014	50.0393	49.3000	1.4774
.400	.400	78.1118	77.5973	0.6587	58.9841	58.4546	0.8892
.498	.462	61.0799	60.2663	1.3320	49.9750	49.1582	1.6344
.510	.528	58.5572	57.7676	1.3484	49.9233	49.1648	1.5193
.400	.600	74.1831	73.7622	0.5674	64.0618	63.6594	0.6281

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Numerical Conformal Transformation of Three-Magnetic-Wall Structures

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Abstract —**The traditional conformal transformation approach for capacitance and impedance evaluation in TEM transmission lines leads to the mapping of the line structure into a rectangular geometry.**

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